

# AI-Aided Signal Reconstruction for Inverse Problems

Workshop “Sensornahe KI/ Sensor AI”

Peter Jung

*[peter.jung@tu-berlin.de](mailto:peter.jung@tu-berlin.de)*

joint work Martin Reiche, Osman Musa and Tom Szollmann



...given the effect...find the cause!

...given the effect...find the cause!

**Goal:** for given observation  $\mathbf{y}$  and forward operator  $\mathbf{A}$ :

$$\text{find } \mathbf{x} \text{ s.t. } \mathbf{A}(\mathbf{x}) \approx \mathbf{y}$$

often solution is ambiguous, priors on  $\mathbf{x}$ , enforce structure!

...given the effect...find the cause!

**Goal:** for given observation  $\mathbf{y}$  and forward operator  $\mathbf{A}$ :

$$\text{find } \mathbf{x} \text{ s.t. } \mathbf{A}(\mathbf{x}) \approx \mathbf{y}$$

often solution is ambiguous, priors on  $\mathbf{x}$ , enforce structure!

- promote structure via regularization
- model parametrization / additional constraints

...given the effect...find the cause!

**Goal:** for given observation  $\mathbf{y}$  and forward operator  $\mathbf{A}$ :

$$\text{find } \mathbf{x} \text{ s.t. } \mathbf{A}(\mathbf{x}) \approx \mathbf{y}$$

often solution is ambiguous, priors on  $\mathbf{x}$ , enforce structure!

- promote structure via regularization
- model parametrization / additional constraints

Examples for explicit (but very simple) structures

- Sparsity/compressibility in some domain
- Low-rankness

→ **well-established theory** (compressed sensing, low-rank recovery, superresolution etc.) with rigorous guarantees

...given the effect...find the cause!

**Goal:** for given observation  $\mathbf{y}$  and forward operator  $\mathbf{A}$ :

$$\text{find } \mathbf{x} \text{ s.t. } \mathbf{A}(\mathbf{x}) \approx \mathbf{y}$$

often solution is ambiguous, priors on  $\mathbf{x}$ , enforce structure!

- promote structure via regularization
- model parametrization / additional constraints

Examples for explicit (but very simple) structures

- Sparsity/compressibility in some domain
- Low-rankness

→ **well-established theory** (compressed sensing, low-rank recovery, superresolution etc.) with rigorous guarantees

**BUT** real data can have complicated structure - AI ?

# Outline

I will discuss some exemplary directions...

- Inverse problems and deep neural networks
- Phase retrieval with deep generative models
- Unrolling of iterative algorithms

# Inverse Problems with Neural Networks



## Inverse Problems / Regularization

Promote desired solutions by selecting an appropriate regularizer

$R : \mathbb{R}^n \rightarrow \mathbb{R}_+$

$$\min_{\mathbf{x}} \quad \|\mathbf{A}(\mathbf{x}) - \mathbf{y}\|_2^2 + \lambda R(\mathbf{x}),$$

## Inverse Problems / Regularization

Promote desired solutions by selecting an appropriate regularizer

$R : \mathbb{R}^n \rightarrow \mathbb{R}_+$

$$\min_{\mathbf{x}} \quad \|\mathbf{A}(\mathbf{x}) - \mathbf{y}\|_2^2 + \lambda R(\mathbf{x}),$$

Examples for  $R(\mathbf{x})$ :

- Tikhonov regularization  $\|\mathbf{W}\mathbf{x}\|_2^2$
- Sparsity w.r.t. some basis/dictionary:  $\|\mathbf{W}\mathbf{x}\|_1$
- Piece-wise constant signal:  $\|\mathbf{x}\|_{\text{TV}}$
- ...

## Inverse Problems / Regularization

Promote desired solutions by selecting an appropriate regularizer

$R : \mathbb{R}^n \rightarrow \mathbb{R}_+$

$$\min_{\mathbf{x}} \quad \|\mathbf{A}(\mathbf{x}) - \mathbf{y}\|_2^2 + \lambda R(\mathbf{x}),$$

Examples for  $R(\mathbf{x})$ :

- Tikhonov regularization  $\|\mathbf{W}\mathbf{x}\|_2^2$
- Sparsity w.r.t. some basis/dictionary:  $\|\mathbf{W}\mathbf{x}\|_1$
- Piece-wise constant signal:  $\|\mathbf{x}\|_{\text{TV}}$
- ...

**But what if desired properties can not be described mathematically?**

## Inverse Problems / Regularization

Assume, we have only an (algorithmic) denoiser  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  satisfying for example

$$f(\mathbf{x} + \boldsymbol{\eta}) \approx \mathbf{x} \quad \text{for } \boldsymbol{\eta} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_n)$$

for the desired class of structured signals  $\mathbf{x}$ .

## Inverse Problems / Regularization

Assume, we have only an (algorithmic) denoiser  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  satisfying for example

$$f(\mathbf{x} + \boldsymbol{\eta}) \approx \mathbf{x} \quad \text{for } \boldsymbol{\eta} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_n)$$

for the desired class of structured signals  $\mathbf{x}$ .

Simple approach, build regularizer:  $R(\mathbf{x}) = \|\mathbf{x} - f(\mathbf{x})\|_p^p$

- theoretical works: *Network Tikhonov - NETT* ([Li et al, 2018])
- but, computing  $\nabla R(\mathbf{x})$  for descent algorithms ?  
...difficult for “algorithmic”  $f$  (maybe numerically or auto-differentiation...)

can we do something **without computing gradients** ?

# Inverse Problems / Regularization

## Regularization by Denoising - RED

If denoiser  $f$  is locally homogeneous, non-expansive and has symmetric Jacobian. Then

$$R_{\text{RED}}(\mathbf{x}) = \mathbf{x}^\top(\mathbf{x} - f(\mathbf{x})) \geq 0$$

and  $\nabla R_{\text{RED}}(\mathbf{x}) = \mathbf{x} - f(\mathbf{x})$  [Romano, 2016 and Reehorst, 2018]

# Inverse Problems / Regularization

## Regularization by Denoising - RED

If denoiser  $f$  is locally homogeneous, non-expansive and has symmetric Jacobian. Then

$$R_{\text{RED}}(\mathbf{x}) = \mathbf{x}^\top(\mathbf{x} - f(\mathbf{x})) \geq 0$$

and  $\nabla R_{\text{RED}}(\mathbf{x}) = \mathbf{x} - f(\mathbf{x})$  [Romano, 2016 and Reehorst, 2018]

- super fast&simply,  $\nabla f(\mathbf{x})$  not needed
- can be used with plug&play algorithms like *ADMM*

# Inverse Problems / Regularization

## Regularization by Denoising - RED

If denoiser  $f$  is locally homogeneous, non-expansive and has symmetric Jacobian. Then

$$R_{\text{RED}}(\mathbf{x}) = \mathbf{x}^\top(\mathbf{x} - f(\mathbf{x})) \geq 0$$

and  $\nabla R_{\text{RED}}(\mathbf{x}) = \mathbf{x} - f(\mathbf{x})$  [Romano, 2016 and Reehorst, 2018]

- super fast&simply,  $\nabla f(\mathbf{x})$  not needed
- can be used with plug&play algorithms like *ADMM*
- use existing denoiser networks like DnCNN [Zhang et al, 2017]
- above conditions rarely satisfied, **but** usually works nonetheless.

**Constrain also signal domain, e.g., by learning from data!!**



## Inverse Problems / Generative Models

Generative models based on *neural networks* work well for learning complicated signal domains

## Inverse Problems / Generative Models

Generative models based on *neural networks* work well for learning complicated signal domains (e.g. *StyleGAN* [Karras et al., 2018])



<https://www.thispersondoesnotexist.com>

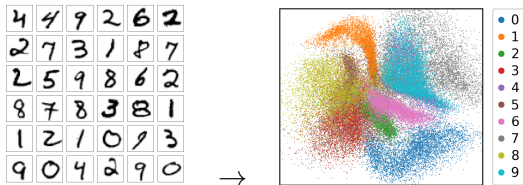
Optimize with relevant signals  $\mathbf{x}$  in the first place.

→ Learn signal distribution from training data

→ yields **generator**  $G : \mathbb{R}^k \rightarrow \mathbb{R}^n$

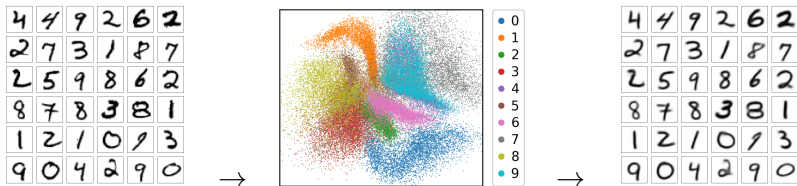
# Inverse Problems / Generative Models

A *Variational Auto-Encoder* [Kingma, 2013] does just that:



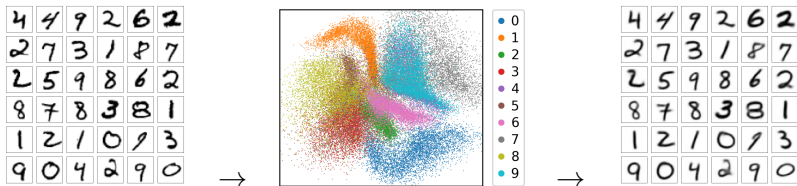
# Inverse Problems / Generative Models

A *Variational Auto-Encoder* [Kingma, 2013] does just that:



# Inverse Problems / Generative Models

A *Variational Auto-Encoder* [Kingma, 2013] does just that:



Take decoder of VAE as signal generator  $G : \mathbb{R}^k \rightarrow \mathbb{R}^n$  and solve

$$\min_{\mathbf{z}} \frac{1}{2} \|\mathbf{A}G(\mathbf{z}) - \mathbf{y}\|_2^2 + \lambda \cdot R(G(\mathbf{z}))$$

latent variable  $\mathbf{z}$ ,  $\mathbf{x} = G(\mathbf{z})$  generated image

# Inverse Problems / Generative Models

Theoretical Guarantees ?

# Inverse Problems / Generative Models

Theoretical Guarantees ?

Theorem ([Bora et al,2017] )

Let  $G : \mathbb{R}^k \rightarrow \mathbb{R}^n$  be a  $d$ -layer feed-forward neural network with ReLU activations and  $\mathbf{A} \in \mathbb{R}^{m,n}$  with  $A_{i,j} \sim \mathcal{N}(0, 1/m)$  where  $m \simeq kd \log n$ .  
Let

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta} \quad \text{for } \mathbf{x} \in \mathbb{R}^n \text{ and noise } \boldsymbol{\eta} \in \mathbb{R}^m.$$

Assume that  $\mathbf{z}^*$  minimizes  $\|\mathbf{A}G(\mathbf{z}) - \mathbf{y}\|_2$  within  $\epsilon$  from the optimum.  
Then with high probability,

$$\|G(\mathbf{z}^*) - \mathbf{x}\|_2 \leq 6 \min_z \|G(\mathbf{z}) - \mathbf{x}\|_2 + 3\|\boldsymbol{\eta}\|_2 + 2\epsilon.$$

- compressed sensing inspired proofs

# Inverse Problems / Generative Models

Theoretical Guarantees ?

Theorem ([Bora et al,2017] )

Let  $G : \mathbb{R}^k \rightarrow \mathbb{R}^n$  be a  $d$ -layer feed-forward neural network with ReLU activations and  $\mathbf{A} \in \mathbb{R}^{m,n}$  with  $A_{i,j} \sim \mathcal{N}(0, 1/m)$  where  $m \simeq kd \log n$ .  
Let

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta} \quad \text{for } \mathbf{x} \in \mathbb{R}^n \text{ and noise } \boldsymbol{\eta} \in \mathbb{R}^m.$$

Assume that  $\mathbf{z}^*$  minimizes  $\|\mathbf{A}G(\mathbf{z}) - \mathbf{y}\|_2$  within  $\epsilon$  from the optimum.  
Then with high probability,

$$\|G(\mathbf{z}^*) - \mathbf{x}\|_2 \leq 6 \min_z \|G(\mathbf{z}) - \mathbf{x}\|_2 + 3\|\boldsymbol{\eta}\|_2 + 2\epsilon.$$

- compressed sensing inspired proofs



# Inverse Problems / Generative Models

Theoretical Guarantees ?

Theorem ([Bora et al,2017] )

Let  $G : \mathbb{R}^k \rightarrow \mathbb{R}^n$  be a  $d$ -layer feed-forward neural network with ReLU activations and  $\mathbf{A} \in \mathbb{R}^{m,n}$  with  $A_{i,j} \sim \mathcal{N}(0, 1/m)$  where  $m \simeq kd \log n$ .  
Let

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta} \quad \text{for } \mathbf{x} \in \mathbb{R}^n \text{ and noise } \boldsymbol{\eta} \in \mathbb{R}^m.$$

Assume that  $\mathbf{z}^*$  minimizes  $\|\mathbf{A}G(\mathbf{z}) - \mathbf{y}\|_2$  within  $\epsilon$  from the optimum.  
Then with high probability,

$$\|G(\mathbf{z}^*) - \mathbf{x}\|_2 \leq 6 \min_z \|G(\mathbf{z}) - \mathbf{x}\|_2 + 3\|\boldsymbol{\eta}\|_2 + 2\epsilon.$$

- compressed sensing inspired proofs
- ☹️ how to compute  $\mathbf{z}^*$  sufficiently accurate (non-convex!) ?

# Inverse Problems / Generative Models

Theoretical Guarantees ?

Theorem ([Bora et al,2017] )

Let  $G : \mathbb{R}^k \rightarrow \mathbb{R}^n$  be a  $d$ -layer feed-forward neural network with ReLU activations and  $\mathbf{A} \in \mathbb{R}^{m,n}$  with  $A_{i,j} \sim \mathcal{N}(0, 1/m)$  where  $m \simeq kd \log n$ .  
Let

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta} \quad \text{for } \mathbf{x} \in \mathbb{R}^n \text{ and noise } \boldsymbol{\eta} \in \mathbb{R}^m.$$

Assume that  $\mathbf{z}^*$  minimizes  $\|\mathbf{A}G(\mathbf{z}) - \mathbf{y}\|_2$  within  $\epsilon$  from the optimum.  
Then with high probability,

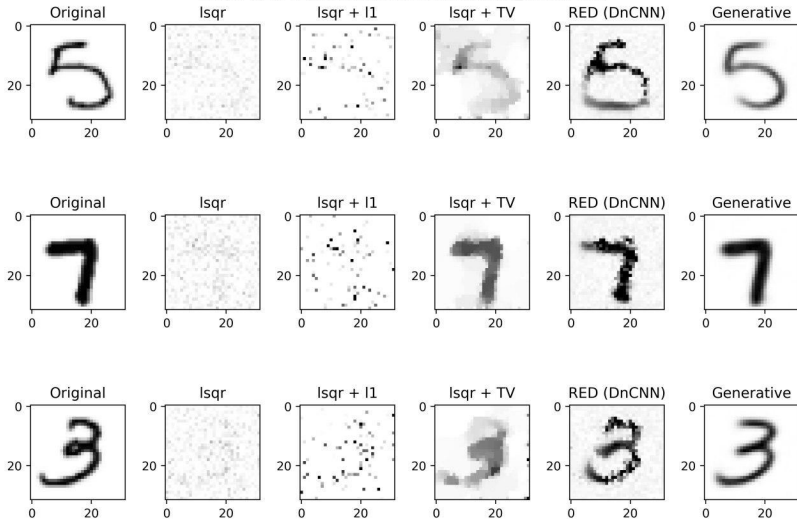
$$\|G(\mathbf{z}^*) - \mathbf{x}\|_2 \leq 6 \min_z \|G(\mathbf{z}) - \mathbf{x}\|_2 + 3\|\boldsymbol{\eta}\|_2 + 2\epsilon.$$

- compressed sensing inspired proofs
- ☹️ how to compute  $\mathbf{z}^*$  sufficiently accurate (non-convex!) ?
- ☹️ undesired dimension scaling is for untrained networks

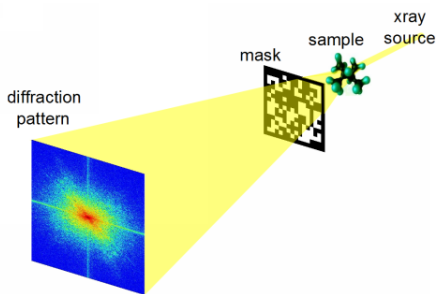
# Inverse Problems / Generative Models

MNIST with Gaussian  $A \in \mathbb{R}^{128 \times 1024}$ , subsampling=1/8

MNIST (32 x 32), measurements: 128, noise\_sigma: 0.01



## Phase Retrieval with Generative Prior



## Phase Retrieval with Generative Prior

Classical problem in physics, engineering and applied math

$$\min_{\mathbf{x}} \|\mathbf{y} - |\mathbf{Ax}|^2\|_2^2 + \lambda R(\mathbf{x})$$

## Phase Retrieval with Generative Prior

Classical problem in physics, engineering and applied math

$$\min_{\mathbf{x}} \|\mathbf{y} - |\mathbf{Ax}|^2\|_2^2 + \lambda R(\mathbf{x})$$

- *nonconvex* but descent methods (like Wirtinger flow) empirically succeed if **initialization is close to optimal value**
- unique solution ?

# Phase Retrieval with Generative Prior

Classical problem in physics, engineering and applied math

$$\min_{\mathbf{x}} \|\mathbf{y} - |\mathbf{Ax}|^2\|_2^2 + \lambda R(\mathbf{x})$$

- *nonconvex* but descent methods (like Wirtinger flow) empirically succeed if **initialization is close to optimal value**
- unique solution ?



## Phase Retrieval with Generative Prior

Classical problem in physics, engineering and applied math

$$\min_{\mathbf{x}} \|\mathbf{y} - |\mathbf{Ax}|^2\|_2^2 + \lambda R(\mathbf{x})$$

- *nonconvex* but descent methods (like Wirtinger flow) empirically succeed if **initialization is close to optimal value**
- unique solution ?
- RED has been proposed as prDEEP  
[Metzler, 2018]
- **solve this problem with as few observations as possible!!**





## Phase Retrieval with Generative Prior

Classical problem in physics, engineering and applied math

$$\min_{\mathbf{x}} \|\mathbf{y} - |\mathbf{Ax}|^2\|_2^2 + \lambda R(\mathbf{x})$$

- *nonconvex* but descent methods (like Wirtinger flow) empirically succeed if **initialization is close to optimal value**
- unique solution ?
- RED has been proposed as prDEEP [Metzler, 2018]
- **solve this problem with as few observations as possible!!**



Use trained network  $G : \mathbb{R}^k \rightarrow \mathbb{R}^n$  [Asim et al., 2019] and [Shamshad et al., 2018]:

$$\min_{\mathbf{z}} \|\mathbf{y} - |\mathbf{AG}(\mathbf{z})|^2\|_2^2 + \lambda \|G(\mathbf{z})\|_{\text{TV}}$$

## Phase Retrieval with Generative Prior

- model error of  $G$  quite high for complicated signal domains
- slow convergence and expensive iterations

## Phase Retrieval with Generative Prior

- model error of  $G$  quite high for complicated signal domains
- slow convergence and expensive iterations

**Hybrid approach:** use it as initialization for a traditional method.

## Phase Retrieval with Generative Prior

- model error of  $G$  quite high for complicated signal domains
- slow convergence and expensive iterations

**Hybrid approach:** use it as initialization for a traditional method.

- 1 few (expensive) descent steps to approach

$$\tilde{\mathbf{z}} \approx \underset{\mathbf{z}}{\operatorname{argmin}} \|\mathbf{y} - |\mathbf{A}\mathbf{G}(\mathbf{z})|^2\|_2^2 + \lambda \|\mathbf{G}(\mathbf{z})\|_{\text{TV}}$$

with randomly initialized  $\mathbf{z}$ .

## Phase Retrieval with Generative Prior

- model error of  $G$  quite high for complicated signal domains
- slow convergence and expensive iterations

**Hybrid approach:** use it as initialization for a traditional method.

- 1 few (expensive) descent steps to approach

$$\tilde{\mathbf{z}} \approx \underset{\mathbf{z}}{\operatorname{argmin}} \|\mathbf{y} - |\mathbf{A}\mathbf{G}(\mathbf{z})|^2\|_2^2 + \lambda \|\mathbf{G}(\mathbf{z})\|_{\text{TV}}$$

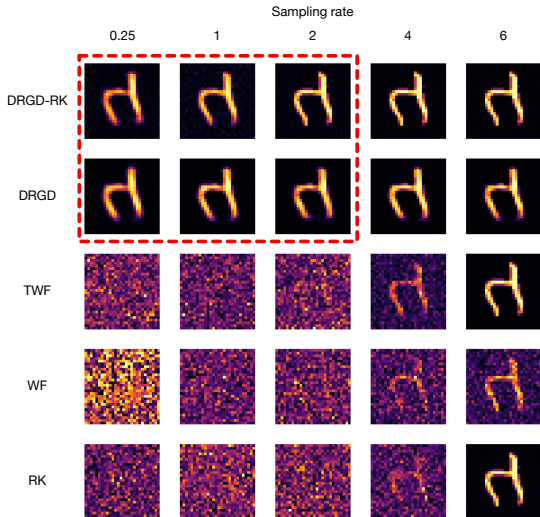
with randomly initialized  $\mathbf{z}$ .

- 2 refine, for  $x \in \mathbb{R}^n$  solve with (superfast) *Randomized Kaczmarz*:

$$\hat{\mathbf{x}} := \underset{\mathbf{x} \in \mathbb{R}^n}{\operatorname{argmin}} \|\mathbf{y} - |\mathbf{A}\mathbf{x}|^2\|_2^2$$

initialized with  $\tilde{\mathbf{x}} = \mathbf{G}(\tilde{\mathbf{z}})$ , overcomes model error of  $G$

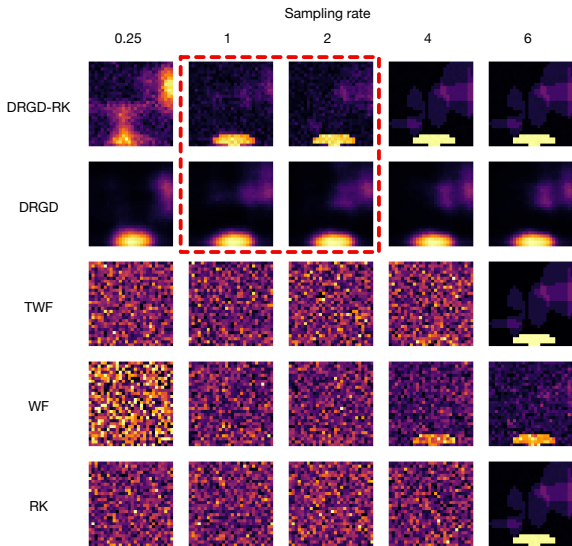
## Results for MNIST



- $\mathbf{y} = |\mathbf{Ax}|^2$
- $\mathbf{A} \in \mathbb{C}^{m \times n}$   
iid. complex normal
- $\mathbf{x} = 28 \times 28$  MNIST
- sampling rate =  $\frac{m}{n}$
- WF=Wirtinger flow
- TWF=truncated WF
- RK=random Kaczmark
- DRGD-RK=deep gradient+RK

same SSIM achieved at 1/6 sampling rate and 1/100 runtime ...

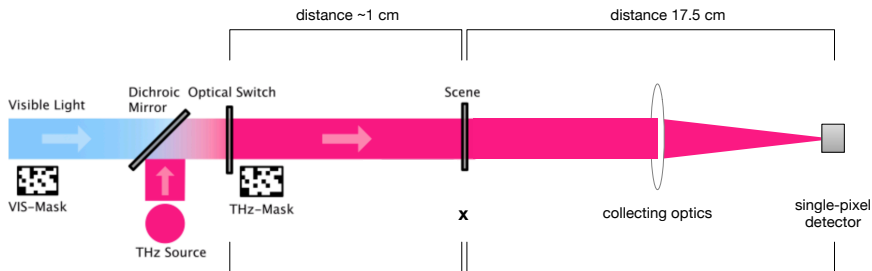
## Results for Shepp-Logan



same SSIM achieved at 1/6 sampling rate and 1/100 runtime ...

# Phase Retrieval for THz Imaging

Cooperation on computational imaging with S. Augustin (DLR/HU)



forward model **A**:

- random iid. binary masks
- discretized diffraction model ( $D_{M \rightarrow S}$  &  $D_{S \rightarrow D}$ )

[Katkovnik et al., 2009]

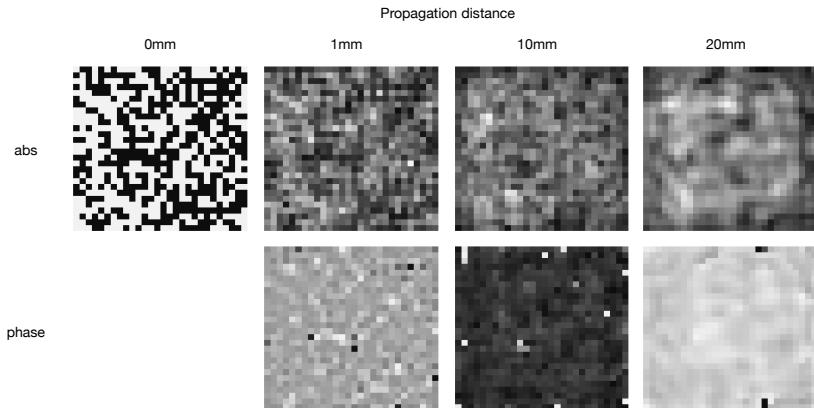


## Phase Retrieval for THz Imaging

effective masks after propagating different (*stand-off*) distances?

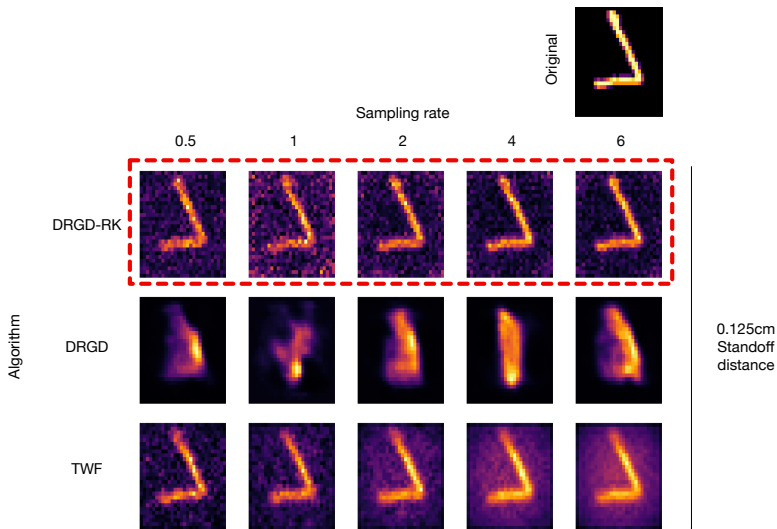
# Phase Retrieval for THz Imaging

effective masks after propagating different (*stand-off*) distances?

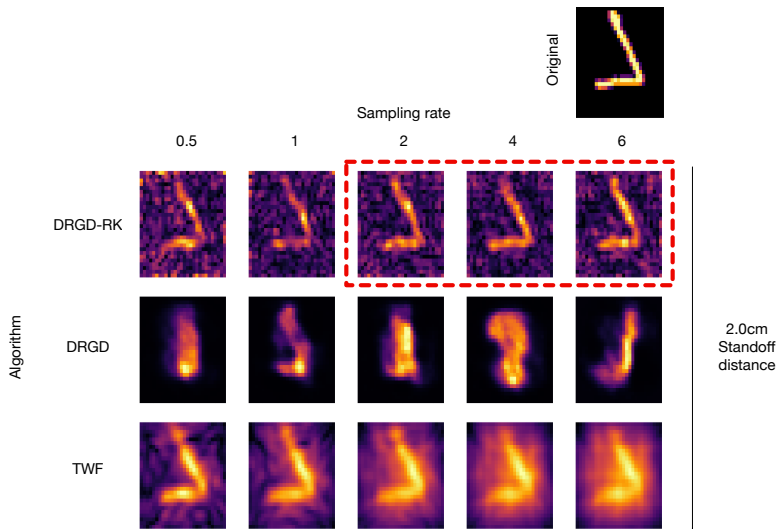


diffraction matrices  $D_{M \rightarrow S}$  loosing rank with increased propagation distance [Katkovnik et al., 2009]

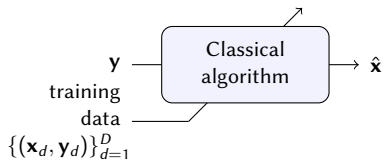
# Results for MNIST at 0.125cm stand-off



# Results for MNIST at 2.0cm stand-off



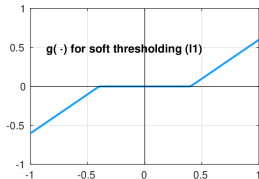
## Unfolding Iterative Algorithms



## Unfolding Algorithms into Networks

- recover a sparse  $\mathbf{x}$  from  $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$  where  $\mathbf{e} \sim N(0, \sigma^2)$
- popular algorithm like ISTA [Daubechies etal, 2004]

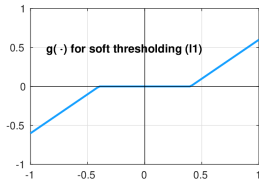
$$\hat{\mathbf{x}}^{t+1} = g(\mathbf{S}\hat{\mathbf{x}}^t + \mathbf{B}\mathbf{y}) \quad \text{with} \quad \begin{aligned} \mathbf{S} &\triangleq \mathbf{I} - \mathbf{A}^T\mathbf{A} \\ \mathbf{B} &\triangleq \mathbf{A}^T \end{aligned}$$



## Unfolding Algorithms into Networks

- recover a sparse  $\mathbf{x}$  from  $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$  where  $\mathbf{e} \sim N(0, \sigma^2)$
- popular algorithm like ISTA [Daubechies etal, 2004]

$$\hat{\mathbf{x}}^{t+1} = g(\mathbf{S}\hat{\mathbf{x}}^t + \mathbf{B}\mathbf{y}) \quad \text{with} \quad \begin{aligned} \mathbf{S} &\triangleq \mathbf{I} - \mathbf{A}^T\mathbf{A} \\ \mathbf{B} &\triangleq \mathbf{A}^T \end{aligned}$$

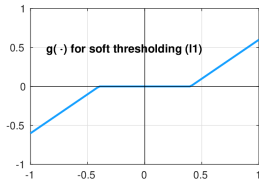


- mismatched prior:  $\mathbf{x} \sim$  Bernoulli-Gaussian (BG) (unknown)

# Unfolding Algorithms into Networks

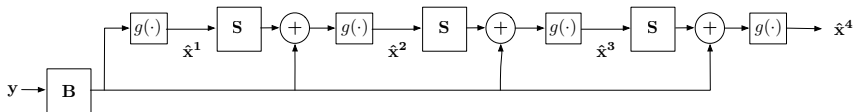
- recover a sparse  $\mathbf{x}$  from  $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$  where  $\mathbf{e} \sim N(0, \sigma^2)$
- popular algorithm like ISTA [Daubechies etal, 2004]

$$\hat{\mathbf{x}}^{t+1} = g(\mathbf{S}\hat{\mathbf{x}}^t + \mathbf{B}\mathbf{y}) \quad \text{with} \quad \begin{aligned} \mathbf{S} &\triangleq \mathbf{I} - \mathbf{A}^T\mathbf{A} \\ \mathbf{B} &\triangleq \mathbf{A}^T \end{aligned}$$



- mismatched prior:  $\mathbf{x} \sim$  Bernoulli-Gaussian (BG) (unknown)

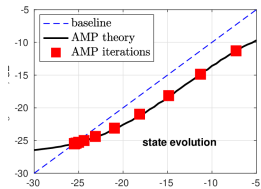
**learned ISTA** (LISTA): unfold iterations as net and "learn" improved parameters using training data [Gregor & LeCun, 2010]





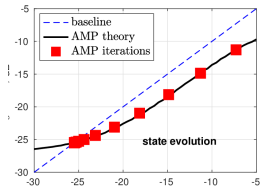
# Unfolding Algorithms into Networks

- *Approximate message passing* (AMP) with Onsager-decoupled iterations [Donoho, Maleki & Montanari 2009], unfold...
- ”learned AMP” (LAMP) [Borgerding & Schniter, 2016]

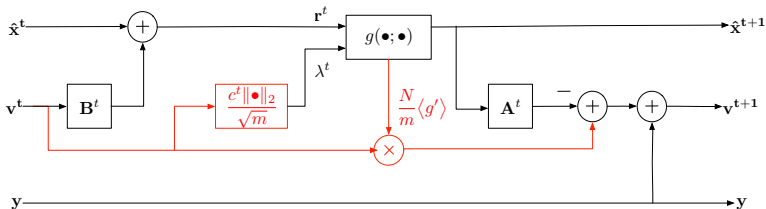


# Unfolding Algorithms into Networks

- *Approximate message passing (AMP)* with Onsager-decoupled iterations [Donoho, Maleki & Montanari 2009], unfold...
- ”**learned AMP**” (LAMP) [Borgerding & Schniter, 2016]

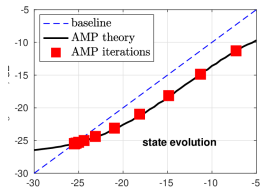


$t^{\text{th}}$  layer: LISTA  $\rightarrow$  **LAMP** (Onsager-decoupled layers)

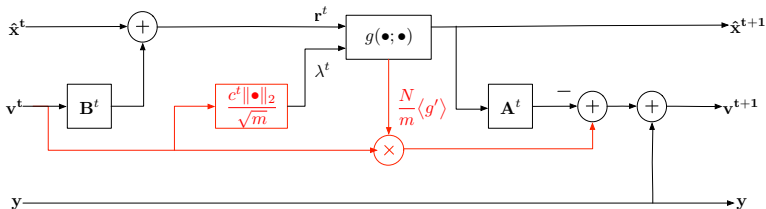


## Unfolding Algorithms into Networks

- *Approximate message passing (AMP)* with Onsager-decoupled iterations [Donoho, Maleki & Montanari 2009], unfold...
- **”learned AMP” (LAMP)** [Borgerding & Schniter, 2016]

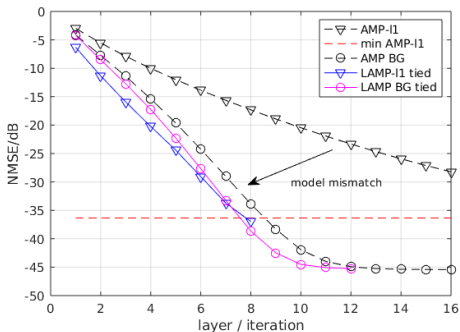
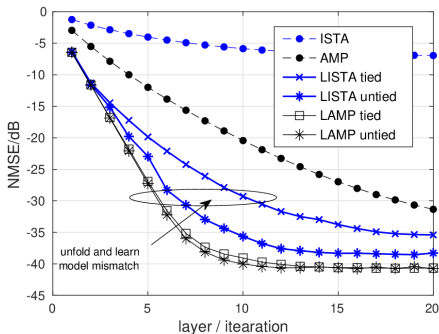


$t^{\text{th}}$  layer: LISTA  $\rightarrow$  **LAMP** (Onsager-decoupled layers)



**can we learn the denoiser matched to unknown prior?**

# Unfolding Algorithms into Networks



- learned algorithms adapt to unknown “prior”
- substantially reduced iteration count

## Conclusion

- neural networks as realistic data priors in recovery algorithms
- ... learned regularizer/ denoiser / proximal mapping
- ... generative model (VAE/GAN etc.)
- helpful in solving challenging problems like *phase retrieval* for realistic data
- ... overcome generator model error by using only as initialization to fast traditional algorithms
- unfolding iterative algorithms and tuning

## Conclusion

- neural networks as realistic data priors in recovery algorithms
- ... learned regularizer/ denoiser / proximal mapping
- ... generative model (VAE/GAN etc.)
- helpful in solving challenging problems like *phase retrieval* for realistic data
- ... overcome generator model error by using only as initialization to fast traditional algorithms
- unfolding iterative algorithms and tuning

**Thank you!**

## References



S. Augustin, P. Jung, S. Frohmann, H.-W. Hübers (2019)

Terahertz dynamic aperture imaging at stand-off distances using a Compressed Sensing protocol

*arXiv 1902.07935v1*



F. Kraemer, Y.-K. Liu (2018)

Phase Retrieval Without Small-Ball Probability Assumptions

*IEEE Transactions on Information Theory* 10.1109/TIT.2017.2757520



E. J. Candes, X. Li, M. Soltanolkotabi (2015)

Phase Retrieval via Wirtinger Flow: Theory and Algorithms

*IEEE Transactions on Information Theory* 10.1109/TIT.2015.2399924



P. Hand, O. Leong, V. Voroninski (2018)

Phase Retrieval under a Generative Prior

*Advances in Neural Information Processing Systems NIPS 31*

## References



T. Karras, S. Laine, T. Aila (2018)

A Style-Based Generator Architecture for Generative Adversarial Networks  
*arXiv 1812.04948*



M. Asim, A. Ahmed, P. Hand (2019)

Invertible generative models for inverse problems: mitigating representation error and dataset bias  
*arXiv 1905.11672*



F. Shamshad, A. Ahmed (2018)

Robust Compressive Phase Retrieval via Deep Generative Priors  
*arXiv 1808.05854*



Y. LeCun (1998)

The MNIST database of handwritten digits  
<http://yann.lecun.com/exdb/mnist/>



# References



A. Chambolle, V. Caselles, M. Novaga, D. Cremers, T. Pock (2009)

An introduction to Total Variation for Image Analysis

*HAL 00437581*



Y. S. Tan, R. Vershynin (2017)

Phase Retrieval via Randomized Kaczmarz: Theoretical Guarantees

*arXiv 1706.09993*



K. Wei (2015)

Solving systems of phaseless equations via Kaczmarz methods: A proof of concept study

*arXiv 1502.01822*



L. A. Shepp, B. F. Logan (1974)

The Fourier reconstruction of a head section

*DOI 10.1109/TNS.1974.6499235*

## References









V. Katkovnik, A. Migukin, J. Astola (2009)

Backward discrete wave field propagation modeling as an inverse problem:  
toward perfect reconstruction of wave field distributions

[http://www.cs.tut.fi/~lasip/DDT/MATRIX\\_DDT.pdf](http://www.cs.tut.fi/~lasip/DDT/MATRIX_DDT.pdf)

## References

-  A. BORA, A. JALAL, E. PRICE, AND A. G. DIMAKIS, *Compressed Sensing using Generative Models*, arXiv e-prints, 2017, arXiv:1703.03208, p. arXiv:1703.03208, <https://arxiv.org/abs/1703.03208>.
-  D. P. KINGMA AND M. WELLING, *Auto-Encoding Variational Bayes*, arXiv e-prints, 2013, arXiv:1312.6114, p. arXiv:1312.6114, <https://arxiv.org/abs/1312.6114>.
-  H. LI, J. SCHWAB, S. ANTHOLZER, AND M. HALTMEIER, *NETT: Solving Inverse Problems with Deep Neural Networks*, arXiv e-prints, 2018, arXiv:1803.00092, p. arXiv:1803.00092, <https://arxiv.org/abs/1803.00092>.
-  S. ANTHOLZER, J. SCHWAB, J. BAUER-MARSCHALLINGER, P. BURGHOLZER, AND M. HALTMEIER, *NETT Regularization for Compressed Sensing Photoacoustic Tomography*, arXiv e-prints, 2019, arXiv:1901.11158, p. arXiv:1901.11158, <https://arxiv.org/abs/1901.11158>.
-  Y. ROMANO, M. ELAD, AND P. MILANFAR, *The little engine that could: Regularization by denoising (RED)*, CoRR, abs/1611.02862 2016, <http://arxiv.org/abs/1611.02862>, <https://arxiv.org/abs/1611.02862>.
-  E. T. REEHORST AND P. SCHNITER, *Regularization by denoising: Clarifications and*