Some Aspects of Weyl-Heisenberg Signal Design in Wireless Commmunication

Peter Jung

peter.jung@tu-berlin.de

Electrical Engineering and Computer Science, TU-Berlin
Communications and Information Theory Chair
TU Berlin, Einsteinufer 25
10587 Berlin, Germany

Motivation

evolution of cellular mobile standards

- 3G (UMTS)
- WiMAX in 2005 (OFDM = some TF signaling)
- 3G HSDPA+ in 2009 ☹
- 4G LTE in 2010 (OFDM) ☹
- 4.5G LTE Advanced in 2016 (OFDM) ☹
- 5G in 2020 ☹/☺ ?

new term: waveforms

- several EU projects (Phydyas, METIS, 5GNow, 5G-Fantastic...)

waveform challenges in 5G

- support robust sporadic communication and asynchronous massive connectivity
- robust against: time&frequency dispersions, asynchronous access, low-cost hardware

⇒ Using Weyl–Heisenberg structures for waveform design

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Outline

Notation and Background

Time–frequency shifts and Spreading representation

- time–frequency shifts $\pi_\mu \in \mathbb{C}^{N\times N}$ for $\mu \in \mathbb{P}_N := \mathbb{Z}_N \times \mathbb{Z}_N$ acts on ONB $\{e_n\}_{n=0}^{N-1}$

$$\pi_\mu e_m := \exp(i2\pi\mu_1 m/N)e_{m\ominus\mu_2}$$

- Weyl commutation rule $\pi_\mu\pi_\nu = \exp(i2\pi[\mu,\nu])\cdot\pi_\nu\pi_\mu$
- $\{\pi_\mu\}_{\mu\in\mathbb{P}_N}$ is ONB for $\mathbb{C}^{N\times N}$ wrt $\langle A,B\rangle = \text{tr}\,(A^*B)/N$.
- spreading representation $\hat{\sigma} = \{\hat{\sigma}_\mu\}_{\mu\in\mathbb{P}_N} \in \mathbb{C}^{N\times N}$ of matrix $H \in \mathbb{C}^{N\times N}$

$$H = \sum_{\mu\in\mathbb{P}_N}\langle\pi_\mu, H\rangle\pi_\mu = \sum_{\mu\in\mathbb{P}_N}\hat{\sigma}_\mu\pi_\mu$$

- symbol $\sigma = \{\sigma_\mu\}_{\mu\in\mathbb{P}_N} \in \mathbb{C}^{N\times N}$ is sympl. Fourier trafo $\sigma = \mathcal{F}_s\hat{\sigma}$ of $\hat{\sigma}$.
- A *considerable simplified* discrete description is:

$$r = Hs + z$$

where $s, r, z \in \mathbb{C}^N$ and channel matrix $H \in \mathbb{C}^{N\times N}$

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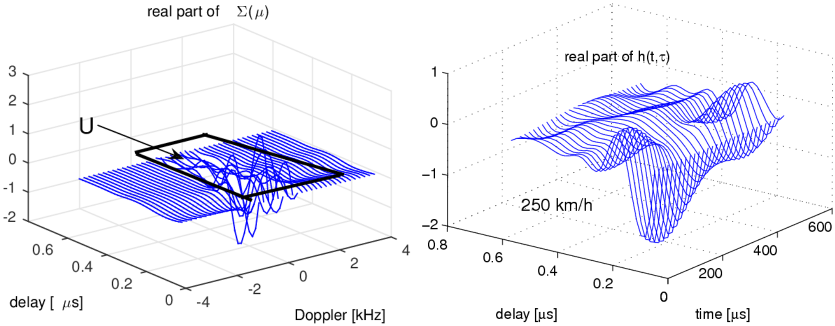
Typical wireless channels

- impulse response $h = F^{-1}\hat{\sigma} \in \mathbb{C}^{N\times N}$ is $h_{t,\tau} = \sum_{\mu_1=0}^{N-1} e^{i2\pi\mu_1 t/N}\hat{\sigma}_{(\mu_1,\tau)}$



- mobile channels are *underspread*: $\text{supp}(\hat{\sigma}) \subset U$ with $|U| \approx 0.01 \times N^2$, and *sparse*
- **but**, poor timing synchronization, oscillator mismatch and phasenoise (“dirty RF”) in multiuser scenarios effectively increase $|U|$.

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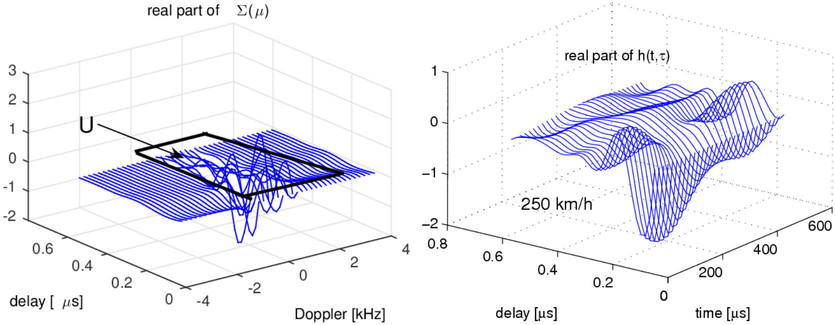

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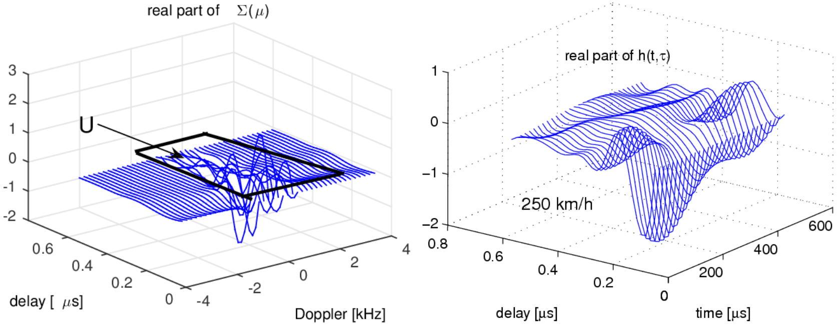

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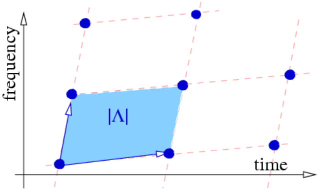
Time–Frequency Signaling

- $\Lambda \subset \mathbb{P}$ subgroup (lattice) generated by $\Lambda = \mathbb{Z}^{2\times 2}$. Define $|\Lambda| = N/\text{card}(\Lambda)$
- For fixed $g, \gamma \in \mathbb{C}^N$ define Gabor sets $\{\gamma_n = \pi_{\Lambda n}\gamma\}$ and $\{g_n = \pi_{\Lambda n} g\}$,



- Transmission of data symbols $\{x_n\} \subset \mathbb{C}$ using N samples $\{y_m\}$ through a channel $\mathcal{H} \in \mathbb{C}^{N\times N}$ with additive noise z:

$$y_m = \langle g_m, \mathcal{H}\gamma_m\rangle x_m + \sum_{n-m\notin \text{SIC}} \langle g_m, \mathcal{H}\gamma_n\rangle x_n + \langle g_m, z\rangle = ☺ + ☹$$

usually without "interference cancellation" $\rightarrow$ SIC $= \{(0,0)\}$.

- TF-density/spectral efficiency is $1/|\Lambda|$.
- OFDM & BFDM: $\Lambda = \text{diag}(T, F)$ & (bi–)orthogonality

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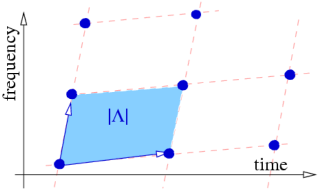

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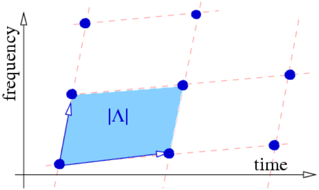

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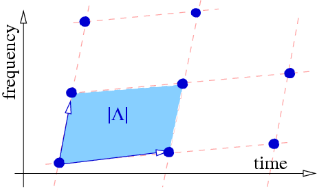

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"signal-to-noise-and-interference ratio"

$$\mathsf{SINR}(g,\gamma,\Lambda) = \mathbb{E}|☺|^2 / \mathbb{E}|☹|^2$$

relation to (g,γ,Λ)*?*

Can we find simple estimates for given scattering scenarios?

WSSUS Pulse Optimization w.r.t. to SINR

- inst. SINR not practicable but second order changes slowly → WSSUS
- $\mathbb{E}(\hat\sigma) = 0$ and $\mathbb{E}(\hat\sigma_\mu \hat\sigma_\nu^*) = C_\mu \delta_{\mu,\nu}$ with scattering prob. $C \in \mathbb{R}_+^{N\times N}$
- $A(X) := \sum_\mu C_\mu \cdot \pi_\mu X \pi_\mu$ for $X \in \mathbb{C}^{N\times N}$
- Weyl–covariant, trace&pos-preserving, $A(\mathrm{Id}) = \mathrm{Id}$, $A(X^*) = A(X)^*$, $A_C \circ A_D = A_D \circ A_C = A_{C*D} \dots$
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- optimize with respect (g,γ) for fixed Λ:

$$\max_{g,\gamma} \mathrm{SINR}(g,\gamma,\Lambda) = \max_\gamma \lambda_{\max}(A(\Pi_\gamma) D(\Pi_\gamma)^{-1})$$

is possible, at least locally, by alternating maximization (initialization, later. . .)

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- $\mathbb{E}(\hat{\sigma}) = 0$ and $\mathbb{E}(\hat{\sigma}_\mu \hat{\sigma}_\nu^*) = C_\mu \delta_{\mu,\nu}$ with scattering prob. $C \in \mathbb{R}_+^{N \times N}$
- $A(X) := \sum_\mu C_\mu \cdot \pi_\mu X \pi_\mu$ for $X \in \mathbb{C}^{N \times N}$
- Weyl–covariant, trace&pos-preserving, $A(\mathrm{Id}) = \mathrm{Id}$, $A(X^*) = A(X)^*$, $A_C \circ A_D = A_D \circ A_C = A_{C*D} \ldots$
- $B(X) := \sum_{\lambda \in \Lambda} \pi_\lambda X \pi_\lambda$ and $D(X) := \sigma_z^2 \mathrm{Id} + (B \circ A)(X)$. Set $\Pi_\gamma = \gamma\gamma^*$:

$$\mathrm{SINR}(g, \gamma, \Lambda) = \frac{\mathbb{E}|☺|^2}{\mathbb{E}|☹|^2} = \frac{\langle \Pi_g, A(\Pi_\gamma) \rangle}{\langle \Pi_g, D(\Pi_\gamma) \rangle}$$

- optimize with respect (g, γ) for fixed Λ:

$$\max_{g,\gamma} \mathrm{SINR}(g, \gamma, \Lambda) = \max_\gamma \lambda_{\max}(A(\Pi_\gamma) D(\Pi_\gamma)^{-1})$$

is possible, at least locally, by alternating maximization (initialization, later. . .)

WSSUS Pulse Optimization, Simplified

- Consider the lower bound:

$$\text{SINR}(g,\gamma,\Lambda) \geq \frac{\mathbb{E}|\smiley|^2}{\sigma_z^2 + B_\gamma - \mathbb{E}|\smiley|^2}$$

with Bessel constant B_γ of $\{\gamma_\lambda\}_{\lambda\in\Lambda}$, i.e., $\|\{\langle\gamma_\lambda, x\rangle_{\lambda\in\Lambda}\}\|_2^2 \leq B_\gamma \|x\|_2^2$

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step ❶ "The TF localization problem"

in general, *"maximize the following term"*

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- TF localization operators[3] $L_{C,\gamma} = \sum_{\mu\in\mathbb{Z}_N^2} C_\mu \langle \pi_\mu\gamma, \cdot\rangle \pi_\mu\gamma$
- without further structure: alternating optimization
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WSSUS Pulse Optimization

step ❶ "Weighted norms of Wigner and Ambiguity Functions"

Some special cases and bounds are known in the continuous setting[4]

Let $A_{g\gamma}(\mu) = \langle g, \pi_\mu \gamma \rangle$,$\|g\|_2 = \|\gamma\|_2 = 1$ and $p, r \in \mathbb{R}$. Furthermore let $C \in L_q(\mathbb{R}^2)$ with $q = \frac{p}{p-1}$. Then for each $p \geq \max\{1, \frac{2}{r}\}$ it holds:

$$\| |A_{g\gamma}|^r C \|_1 \leq \left(\frac{2}{rp} \right)^{\frac{1}{p}} \|C\|_{\frac{p}{p-1}}$$

- equality only possible for Gaussians (g, γ, C), for $C(\mu) = \alpha e^{-\alpha\pi|\mu|^2}$ with $\alpha \geq \frac{2-r}{2}$

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- $U \subset \mathbb{R}^2$, $0 < |U| < \infty$ and $C = \chi_U / |U|$. With $r^* = \max\{r, 2\}$:

$$\| |A_{g\gamma}|^r C \|_1 < \begin{cases} e^{-\frac{r|U|}{2e}} & |U| \leq 2e/r^* \\ \left(\frac{2}{r^*|U|} \right)^{r/r^*} & \text{else} \end{cases}$$

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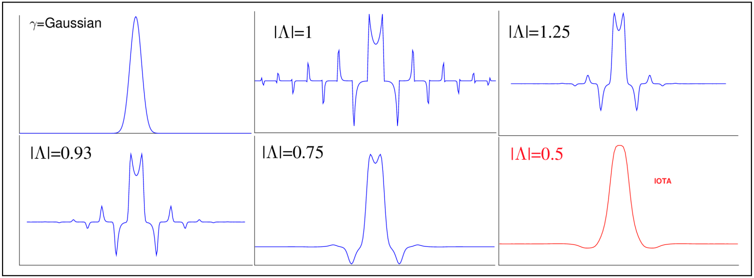
"Balian–Low Theorem" [6]



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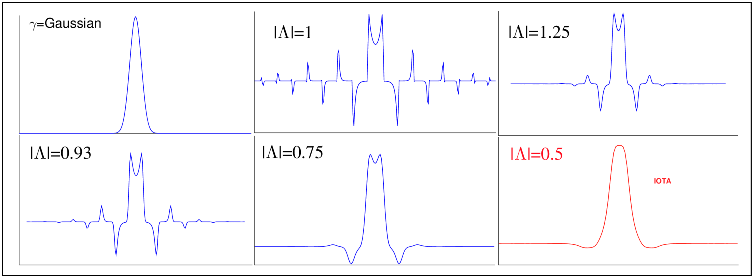

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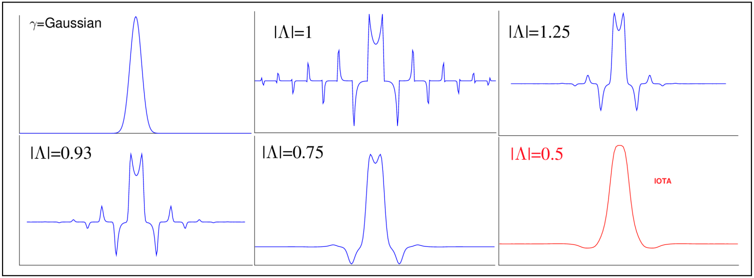

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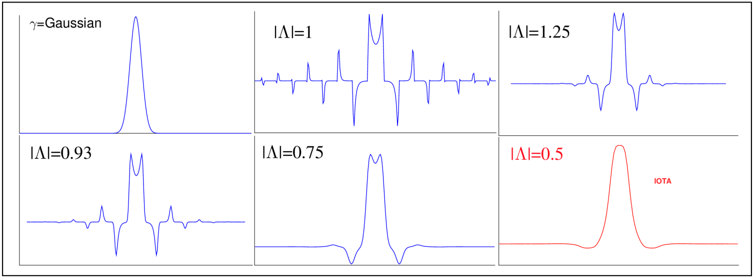

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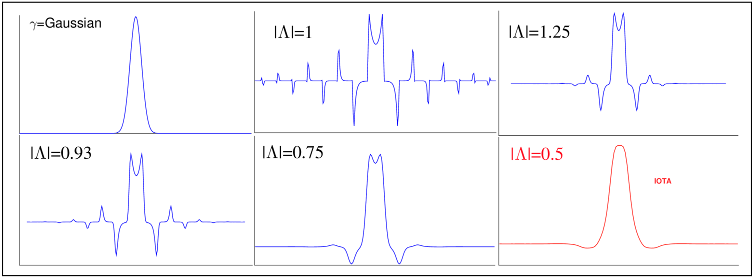

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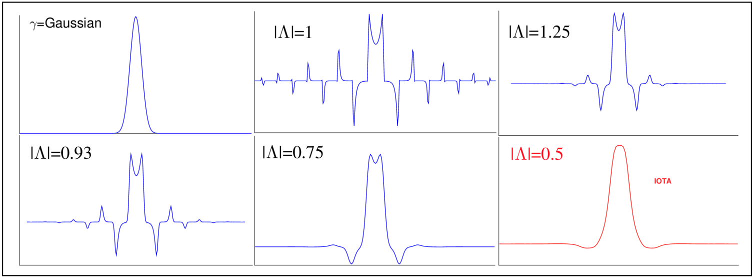

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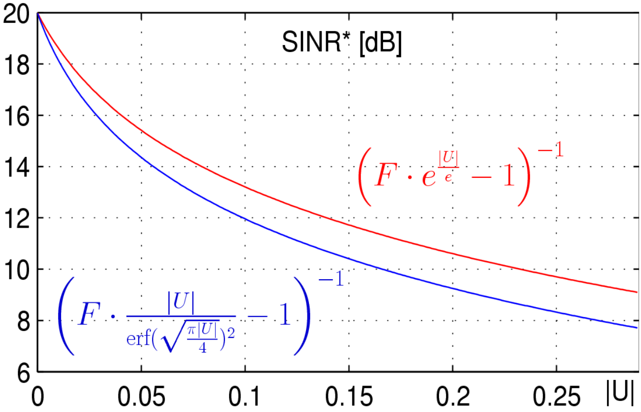
WSSUS Pulse Optimization, 2-Step Strategy

"without Balian-Low..."

Let $F = \sigma_z^2 + |\Lambda|^{-1}$. For the optimal signaling we would get then:

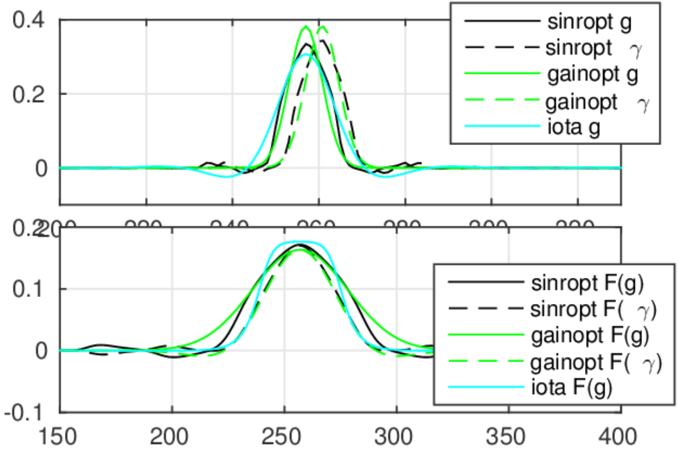


- Upper bound holds for any shape of U
- Lower bound is for $U = \square$ and ignoring the Balian-Low Theorem

Numerical Experiments

- alternating max. $\text{SINR}(g, \gamma, \Lambda) = \mathbb{E}|☺|^2/\mathbb{E}|☹|^2$, initialized by lower bound
- alternating max. $\mathbb{E}|☺|^2$ *without* min. B_γ, initialized by lower bound
- alternating max. $\mathbb{E}|☺|^2$ *and* min. B_γ, initialized by lower bound
- matched Gaussians
- matched tight frame from Gaussians (IOTA)

Numerical Experiments, Example



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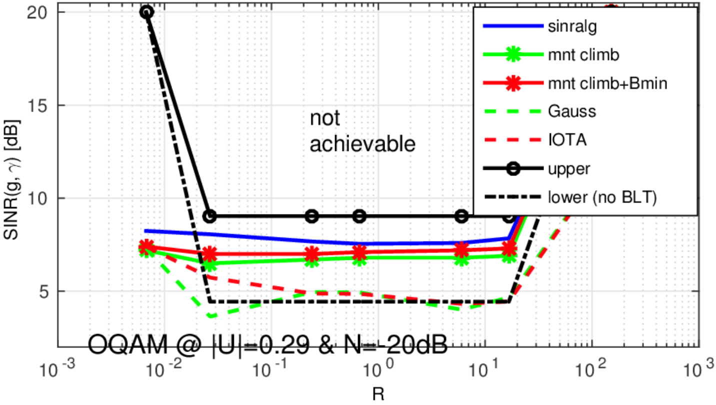

- design on $L = 512$ sample values at bandwidth W
-

delay spread $\tau := \tau_d \cdot W$	0	1	5	9	29	49	149
Doppler $B := B_D/W \cdot L$	74	37	12	7	2	1	0

- $(\tau + 1)(2B + 1) = 150$ ($|U| \approx 0.29$) fixed and $R = \frac{\tau+1}{2B+1}$ as in the table
- OQAM modulation at $|\Lambda| = 1/2$.

Summary and Conclusions

summary:

- waveform optimization is again a hot topic for 5G
- bounds to characterize TF localization

open problems:

- Using group structure& majorization arguments&symmetry of C to charactize:

$$\sum_{\mu\in\mathbb{Z}_N^2} C_\mu |\langle g, \pi_\mu \gamma\rangle|^2 \geq |\langle g, \sum_{\mu\in\mathbb{Z}_N^2} C_\mu \pi_\mu \gamma\rangle|^2$$

  using finite-dim. equivalents of Gaussians
- Finite and quantitative versions of the Balian-Low theorem
- How to solve numerically efficient:

$$\max_{\mathrm{tr}\,(X)=1, X\geq 0} \|A(X)D(X)^{-1}\| = \max_{g,\gamma} \frac{\mathbb{E}|☺|^2}{\mathbb{E}|☹|^2}$$

- Include the lattice Λ

$$\max_{g,\gamma,\Lambda} \frac{1}{|\Lambda|} \log(1 + \frac{\mathbb{E}|☺|^2}{\mathbb{E}|☹|^2})$$

Summary and Conclusions

summary:

- waveform optimization is again a hot topic for 5G
- bounds to characterize TF localization

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